ANNOTATION

of dissertation for the Philosophy Doctor (Ph.D.) degree on «6D060500- Nuclear Physics» specialty

Mansurova Aizhan

Gravitational field of neutron stars considering the quadrupole moment

The general characteristics of the study

This work is focused on explaining the gravitational field of astrophysical compact objects, such as white dwarfs and neutron stars using Einstein's theory of general relativity.

It investigated interior solutions of Einstein's equations in the case of static and axially symmetric perfect fluid spacetimes.

It imposed the physical condition that the interior spacetime can be matched smoothly with the exterior q- metrics characterized by two parameters, which determine the mass and quadrupole moment of the source. In this work, it was assumed that the interior counterpart of the exterior q-metric is described by an isotropic perfect fluid, for the sake of simplicity. Indeed, the most general interior solution, which is compatible with an exterior static and axisymmetric gravitational source, accepts up to four stresses. Thus, we are assuming here that three of them are negligibly small so that we end up with a perfect fluid with only one isotropic stress. Moreover, the isotropic perfect fluid is a very idealized model since it has been shown that static and isotropic perfect fluid sources must be spherical, at least in the case of an incompressible equation of state. This is related to the fact that even small pressure anisotropies can generate breaks in the fluid distribution, drastically moving away from the idealized isotropic configuration. Furthermore, analyzing the stability of the isotropy condition, it has been established recently that the real physical processes that occur during the stellar evolution inevitably lead to the appearance of pressure anisotropies, which cannot disappear during the dynamic evolution of the mass distribution. Consequently, several results indicate that the final equilibrium configuration of a realistic stellar evolution is characterized by the presence of pressure anisotropies. Nevertheless, in this work, we assume the isotropy condition to simplify the mathematical complexity of the resulting field equations. The results show that it is possible to find approximate perfect fluid solutions with quadrupole, which are consistent from a mathematical point of view, in the sense that Einstein's equations are satisfied for certain equations of state.

The actuality of the study

The theory of gravitation proposed by Einstein is widely used to explain the behavior of the gravitational field, as its validity has been experimentally proven in a wide range of macroscopic scenarios. To determine the gravitational field generated by compact objects such as white dwarfs, neutron stars, and planets, two related problems must be considered: the interior and exterior gravitational fields. Recent research has suggested that higher multipole moments may play a significant role in

accurately describing the gravitational field of such objects and these efforts are crucial in advancing our understanding of the behavior of these objects in space.

However, the search for physically relevant interior solutions is not an easy task. The difficulties increase once we consider the nonuniqueness of the solutions. Indeed, whereas the Birkhoff theorem guarantees that the Schwarzschild metric is the only spherically symmetric vacuum solution of Einstein's equations, there exist many spherically symmetric interior solutions that can be matched with the Schwarzschild metric. In the case of axial symmetry, the situation is even more complicated. In the recent work, several exterior solutions with quadrupoles were compared. It was found that they are all represented by diverse analytical expressions and are characterized by different sets of multipole moments. In this sense, they are all physically different from each other. One can, therefore, expect that there will be many interior metrics that can be matched with each one of the exterior solutions. One example of this situation is given by a recently proposed interior solution for the q-metric19 and the solutions that we will analyze in this work.

The purpose of the research

In order to investigate the interior Einstein's equations in the case of a static, axially symmetric, perfect fluid source we investigate the interior and exterior gravitational field of neutron stars with quadrupole momentum.

To achieve this purpose, the following tasks were set:

1. To calculate field equations that take into account the quadrupole moment to explain the gravitational field of compact objects

2. To describe the gravitational field of white dwarfs and neutron stars, taking into account the obtained field equations, equations of state of various types, and matching conditions with the external gravity field of stars

3. To analyze the equations of state describing the gravitational field of white dwarfs and neutron stars taking into account the quadrupole moment and determining the effective equations of state

Objects of research:

Compact objects such as white dwarfs and neutron stars

The subject of study:

The subjects of the research are the analytical and numerical calculation methods in special software.

The main provision for the defense

1. The external static solutions of Einstein's field equations, which consider quadrupole momentum, are determined by the group of 5-parametric solutions, and in this case, the mentioned solutions include both asymptotically smooth solutions and non-asymptotically smooth solutions.

2. Einstein's field equations, which consider quadrupole momentum, For the internal gravitational field of compact objects with variable density satisfy the physical conditions such as mass distribution, mass boundary, the radius of a compact object and its energy, identity and uniformity consistent with an external approximate metric on the surface of a deformable object.

3. The density-dependent effective pressure equation of state for neutron stars can be approximated by the equation of state of the polytrope and solutions of corresponding field equations have no singularity inside neutron stars.

Scientific novelty

1. A class family of static exterior solutions of Einstein's field equations with quadrupole was found. The general approximate exterior solution with quadrupole moment is represented by a 5-parameter family of solutions. In this particular case, these solutions contained asymptotically flat solutions, and non-asymptotically flat solutions are also contained, it indicated that the solutions are more general. In addition, it is found the metric functions as the Newtonian limit of general relativity with quadrupole therefore, the solutions can be used to describe the exterior gravitational field of deformed mass distribution.

2. New non-spherical solutions of Einstein's equations, with quadrupole momentum and non-constant density, were obtained for the first time and satisfy the following physical conditions: inside the mass distribution, as well as the conditions for limiting values of the mass, the radius of the compact object and positive energy; also, satisfy the matching conditions: smoothly matched with the exterior approximate metric at the surface of the deformed object. These results show that it is possible to find such solutions, implying that the approximate approach is compatible with the imposed physical conditions.

3. It was found that the effective pressure-density-dependent equation of state can be approximated by the polytropic equation of state for neutron stars. The internal solutions corresponding to the external metric with the singular property satisfied the energy conditions of density and pressure. It was found that the obtained solutions can be used to describe the gravitational field for compact objects, taking into account the quadrupole moment.

Method of research

For the theoretical studies, it was mainly used the methods of general relativity in order to constrain the parameters of studied models and scenarios. The proposed results were tested numerically using advanced numerical techniques and computer simulations. Further, obtained analytical and numerical results were compared with the results of research from specialized journals and articles about compact astrophysical objects, such as neutron stars and white dwarfs.

The scientific and practical significance of the work

This research work concerns the theoretical investigation. The results which were found can allow one to derive the value of the quadrupole and could lead to important applications in astrophysics, navigation systems, and satellite technology, where relativistic effects are expected to play a nonnegligible role.

The personal contribution of the author

The personal contribution of the author is that the author was directly involved in obtaining the main scientific results. All the results of theoretical calculations and numerical analysis of models were obtained personally by the author. The setting of tasks and the development of ideas were carried out in collaboration with scientific consultants. All publications on the topic of the thesis were prepared with his direct participation.

Reliability of results

1. The theoretical assumptions described in this work are the continuation of the results of well-known scientific works in the field of general relativistic theory. The results obtained are associated with the results of the works of previous researchers and are their logical continuation.

2. The theoretical results given in the thesis are in good agreement with the results of the previous works.

Approbation testing of thesis

The main results of the thesis were presented and discussed at the seminars of the Physics and Technology Faculty of the Al-Farabi Kazakh National University, as well as at the following international conferences:

1. International scientific conference of students and young scientists "Farabi alemi" (2018, Almaty, Kazakhstan);

2. International scientific conference of students and young scientists "Farabi alemi" (2019, Almaty, Kazakhstan);

3. International scientific conference of students and young scientists "Farabi alemi" (2020, Almaty, Kazakhstan);

Publication:

According to the materials of the dissertational work, 7 publications were published. Four works among these publications are articles, one of them has citations in Thomson Reuters (ISI Web of Knowledge, Thomson Reuters) and Scopus databases, three articles in scientific publications recommended by the Committee on the Control of Education and Science of the Ministry of Education (CCESME), and Science of the Republic of Kazakhstan, three reports at local international conferences.

1. Abishev M. et al. Approximate perfect fluid solutions with quadrupole moment //International Journal of Modern Physics D. – 2021. – Т. 30. – №. 13. – С. 2150096. (Scopus: highest percentile 95%; percentile in the thesis' field: 70%)

2. Abishev M. E. et al. Стационарное вакуумное решение уравнений Эйнштейна //Вестник. Серия Физическая (ВКФ). – 2019. – Т. 69. – №. 2. – С. 4-9. (recommended by the CCESME)

3. Абишев М. Е. и др. Определения релятивистских мультипольных моментов в ньютоновской гравитации массивных объектов //Вестник Казахского национального университета. Серия физическая. – 2020. – №. 1. – С. 11-18. (recommended by the CCESME)

4. Mansurova A. A. et al. Согласование условий для внутреннего и внешнего пространства-времени астрофизических компактных объектов //Вестник. Серия Физическая (ВКФ). – 2019. – Т. 71. – №. 4. – С. 45-50. (recommended by the CCESME)

5. А. А. Мансурова, Studies of physics properties of neutron stars//International Scientific Conference of Students and Young Scientists «Farabi Alemi», April 9- 12, 2018, Almaty, Kazakhstan, Book of Abstracts, ed. by A. E. Davletov (Almaty, 2018), p. 46. (Local International Conference)

6. Mansurova A., Alimkilova M. Description of the quadrupolar mass distribution with stationary generalized q-metric//International Scientific Conference of Students and Young Scientists «Farabi Alemi», April 8- 11, 2019, Almaty, Kazakhstan, Book

of Abstracts, ed. by A. E. Davletov (Almaty, 2019), p. 19. (Local International Conference)

7. Мансурова А. А., Бейсен Н. А., Иващиук В. Д. Исследование условий сопоставления общей теорий относительности// Международная конференция студентов и молодых ученных «Фараби әлемі», April 6- 9, 2020, Алматы, Қазақстан, Книга аннотаций под ред. Абишева М. Е. (Алматы, 2020), с. 19. (Local International Conference)

Structure and volume of the dissertation

The dissertation consists of an introduction, four chapters, conclusions and a list of references. It is presented on 101 pages of typewritten text and contains 20 figures and 97 references.